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Use of Constructed Variables in the Method of Strained Coordinates

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In his book, Van Dyke[1] recommends that the method of strained coordinates, or the PLK method,[2] not be used with a "constructed" dependent variable, such as the velocity potential. The purpose of this Note is to clarify this recommendation and to demonstrate that a constructed dependent variable can be used, as long as the straining is determined by recourse to the proper physical variable.

As an example, consider the problem statement for supersonic flow past a thin airfoil (cf., Chap. 6 of Ref. 1). With the notation of Ref. 1, the problem statement for a uniform first approximation in terms of the velocity potential reads

$$\phi_{\xi\eta}+(\gamma+1)M^4\phi_\xi\phi_{\xi\xi}/2B^2=0 \tag{1}$$

Here M is the Mach number, $B=(M^2-1)^{1/2}$, and ξ and η "semicharacteristic" coordinates $x-By$ and By, respectively. The coordinate x is in the streamwise direction, and y is perpendicular to it. With the airfoil surface defined by $y=\epsilon T(x)$, and with ϵ a small parameter, the boundary conditions are given by

$$\phi_\xi(\xi,0)=-\epsilon T'(\xi)/B, \quad \phi=0 \text{ upstream} \tag{2}$$

A perturbation series is then defined as

$$\phi(\xi,\eta;\epsilon)\sim\epsilon\phi_1(s,t)+\epsilon^2\phi_2(s,t)+\ldots \tag{3}$$

and the coordinate straining is given by

$$\xi\sim s+\epsilon\xi_2(s,t)+\ldots, \quad \eta=t \tag{4}$$

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After substitution for $\phi(\xi,\eta;\epsilon)$ from Eq. (3) and use of Eqs. (4) to relate ξ and η derivatives to s and t derivatives, Eq. (1) becomes

$$\epsilon\phi_{1st}+\epsilon^2\phi_{2st}-\epsilon^2(\xi_{2s}\phi_{1st}+\xi_{2st}\phi_{1s}+\xi_{2t}\phi_{1ss}) + (\gamma+1)M^4\epsilon^2\phi_{1s}\phi_{1ss}/2B^2=0 \tag{5}$$

The first approximation is therefore given by solution of the equation

$$\phi_{1st}=0 \tag{6}$$

with boundary conditions obtained from a similar treatment of Eqs. (2)

$$\phi_{1s}(s,0)=-T'(s)/B, \quad \phi_1=0 \text{ upstream} \tag{7}$$

The solution for ϕ_1 is

$$\phi_1(s,t)=-T(s)/B \tag{8}$$

and the differential equation for ϕ_2 becomes

$$\phi_{2st}=-\frac{1}{B}\frac{\partial}{\partial s}\left\{\xi_{2t}T'(s)+\frac{\gamma+1}{4}\frac{M^4}{B^3}[T'(s)]^2\right\} \tag{9}$$

If the straining rule is now applied to the velocity potential, ξ_2 can be chosen as so to eliminate the right-hand side of Eq. (9), thereby ensuring that ϕ_2 be no more singular than ϕ_1. This yields

$$\xi_{2t}=-(\gamma+1)M^4T'(s)/4B^3 \tag{10}$$

which is precisely half the "correct" straining that would be obtained if the problem were posed in terms of the velocity $u=\phi_\xi$ (Ref. 1).

It must be noted, however, that the straining rule in these two instances has been applied to different quantities. If the straining rule is applied by choosing ξ_2 so that the second approximation for the velocity is no more singular than the first, the results will not be the same. Integration of Eq. (9) yields

$$\phi_2(s,t)=-\{\xi_2T'(s)+(\gamma+1)M^4t[T'(s)]^2/4B^3\}/B + g(t)+k(s) \tag{11}$$

where $g(t)$ and $k(s)$ are functions of integration. With the velocity given as

$$u(\xi,\eta;\epsilon)=\phi_\xi=\epsilon u_1(s,t)+\epsilon^2u_2(s,t)+\ldots \tag{12}$$

the second approximation for the velocity is

$$u_2(s,t)=\phi_{2s}-\xi_{2s}\phi_{1s} \tag{13}$$

Use of Eqs. (8) and (11) for ϕ_1 and ϕ_2 yields

$$u_2(s,t)=-T''(s)[\xi_2+(\gamma+1)M^4tT'(s)/2B^3]/B+k'(s) \tag{14}$$

If ξ_2 is now chosen so that u_2 is no more singular than u_1 as $t\to\infty$,

$$\xi_2(s,t)=-(\gamma+1)M^4tT'(s)/2B^3+f(s) \tag{15}$$

and $f(s)=0$ so that the straining vanishes at $y=0$. Equation (15) reproduces Eq. (6.36) in Ref. 1, which is the "correct" straining. Hence the proper straining can be obtained as long as the straining rule is applied to the same quantity; whether a physical variable or a constructed variable precedes this step is immaterial.

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Improved Measurement of Turbulent Intensities by Use of Photon Correlation

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Introduction

MEASUREMENTS of turbulent intensities exceeding 30% are now realizable using a laser velocimeter and frequency shifting device such as a Bragg cell. In addition, by using the photon correlation processing scheme, the turbulent measurements can be made in complex flowfields with recirculation regions.[1] The photon correcting techniques offer improved sensitivity compared to frequency counter and tracker processes by permitting velocity measurements to be made even when insufficient signal photons are available to define the classical scattering signal. Naturally occurring contaminant particles are used as scatterers with no artificial seeding introduced.

One of the primary disadvantages of the photon correlation scheme is that all of the turbulent information must be obtained from the autocorrelation function of the fluctuating velocity field. In fact, the location and relative amplitude of the first several maxima and minima are necessary to determine the local mean velocity and the local turbulent intensity. If the autocorrelation function is skewed or distorted for any of several often occurring reasons, the credibility of the data obtained is then open to challenge.

The purpose of this Note is to describe one approach to improving the accuracy of the turbulent information obtained from the autocorrelation function. Polynomial curve fitting and then averaging is used to eliminate the function distortion.

Equipment and Experimental Procedure

This experiment is performed in the near field region of a two-dimensional turbulent wake flow. A cylinder 11.75 cm in diameter is placed in the test section whose dimensions are 7.0 cm wide by 150.0 cm long. The tunnel flow is kept at a constant velocity of 5.9 m/s with the Reynolds number equal to 48,000 based on the cylinder diameter. Measurements are made at several downstream locations.

The optical arrangement is shown in Fig. 1. A lens with a focal length of 100 cm is used with the beam separation equal to 2 cm. The laser used is a 15 mW helium neon type. Careful alignment of the photomultiplier tube is performed in order to achieve a maximum signal level.

The signal from the photomultiplier tube is passed directly to a Malvern correlator. The correlator is used to determine the autocorrelation functions of the turbulent velocity field.

The power spectra as found by Lumley et al.[2] is

$$P(w) = \tfrac{1}{4}[\exp\{-(w+w_0)^2 a^2/2\} + \exp\{-(w-w_0)^2 a^2/2\} \tag{1}$$

where w_0 is the Doppler frequency, and a is a constant.[3] Taking the Fourier transform yields the autocorrelation function

$$R(t) = \exp(-t^2/2a^2)\cos(w_0 t) \tag{2}$$

Consider the case when the autocorrelation curve has been distorted by either a lack of number of fringes in the control volume or significant background scattered light (Fig. 2). The calculation of the turbulent intensity from an autocorrelation function can be approximated by determining the decrease in the amplitude of the cosine-like wave. One approximation[4] used is the following

$$\frac{u_{\rm rms}}{U} = \frac{1}{\pi}\left(\tfrac{1}{2}(R-1) + \frac{1}{2n^2}\right)^{1/2} \tag{3}$$

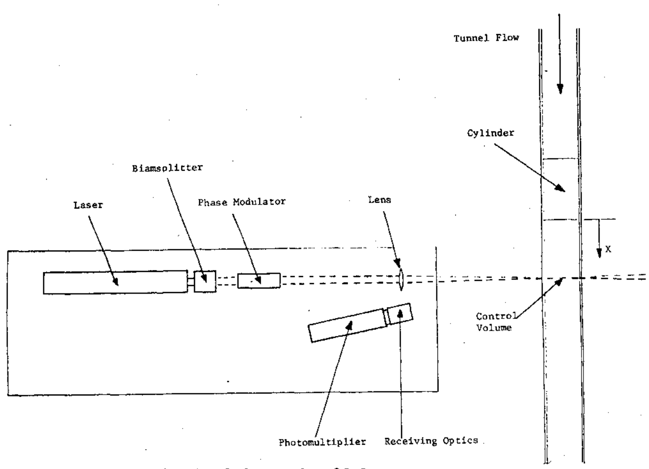

Fig. 1 Schematic of laboratory setup.

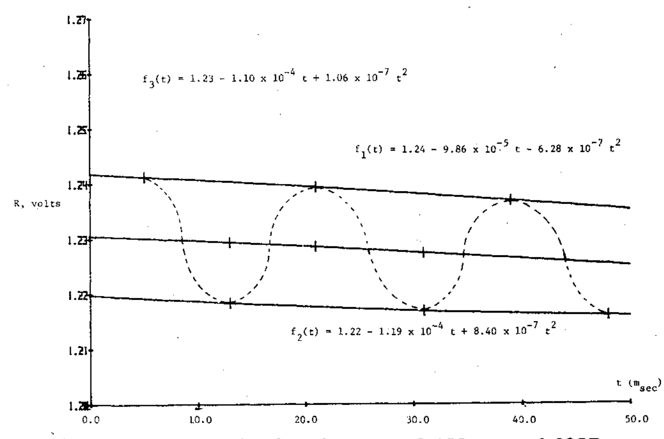

Fig. 2 Autocorrelation function, $x = -2.15D$, $y = -0.085D$.

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